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N.L.O. Corrections to Multiple Jet Cross sections

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ABSTRACT

A general method is outlined to calculate next to leading order corrections to cross sections involving multiple jet final states. The method uses compact universal building blocks for initial and final-state radiation, and is applicable to a wide variety of processes. This modularity removes the usual technical problems in these types of calculations and opens the way for the calculation of the radiative corrections to high multiplicity jet cross sections.

1 Introduction

A good understanding of final states with a definite number of jets is becoming more and more important. This is because demanding jets in the final state is a powerful method of reducing the standard QCD backgrounds to potential signals of new physics.

A good example is the top search in the single charged lepton channel. By demanding four jets in the final state in addition to the lepton, the standard QCD background is greatly reduced. In fact, the four jet plus lepton cross section from the top quark pair decay is larger than the background for a top quark mass between 100 GeV and 150 GeV making it possible to look for the top quark in this final state [1].

In order to use jet final states as an analysis tool the effects of radiative corrections have to be understood. While the methods for calculating leading order cross sections for multijet final states are very well understood [2], next to leading order calculations are not as well developed.

Adding next to leading order corrections gives two important improvements over the leading order calculations. First, the cross sections become dependent on the jet clustering algorithm, while soft radiation outside the jet cones and radiation outside the detector is simulated. To make full use of these improvements it is essential that the hard phase space integrals are performed numerically with the aid of a Monte Carlo. This enables one to adapt the calculation to a particular experiment with its specific detector properties, jet definitions and analysis cuts.

Second, including the next to leading order correction reduces the theoretical uncertainties due to the choice of the renormalization and factorization scales, and makes the prediction for the total cross section more reliable.

2 Theory

The methods for performing the hard phase space integrals numerically are well-known: one introduces a theoretical resolution parameter which isolates the soft and collinear radiation. These two divergent contributions are calculated analytically and added to the virtual corrections thereby canceling the soft and collinear divergences. This introduces an explicit logarithmic dependence on the resolution parameter which, in principle, is canceled by adding the hard phase space above the resolution cut. This can be done numerically using a Monte Carlo approach allowing the incorporation of experimental constraints.

This method has been developed to its fullest in QED for including photon radiation [3]. Due to the factorization in the soft photon limit of the cross section into an eikonal factor containing the soft singularity multiplied by the hard cross section it can be set up quite generally. Only the trivial eikonal factor has to be regularized, while the hard cross section can be calculated in the

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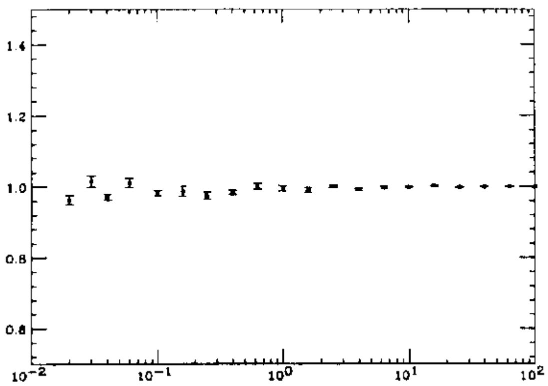

Figure 1: s_{min} dependence (in GeV^2) of the $p\bar{p} \to W^{\pm} + 0$ jet cross section at $\mathcal{O}(\alpha_s)$ as a function of $\sigma(s_{min})/\sigma(s_{min} = 100\,\mathrm{GeV}^2)$ using the loose CDF cuts as described in the text.

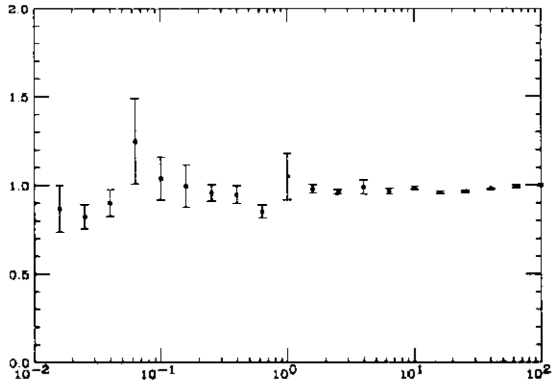

Figure 2: s_{min} dependence (in GeV^2) of the $p\bar{p} \to W^{\pm} + 1$ jet cross section at $\mathcal{O}(\alpha_s)$ as a function of $\sigma(s_{min})/\sigma(s_{min} = 100\,\mathrm{GeV}^2)$ using the loose CDF cuts as described in the text.

usual way.

In QCD the situation is more complicated since although collinear radiation can be factorized into a singular splitting function times a hard cross section, there is no analogous soft gluon factorization because the gluons themselves carry color charge. Furthermore, for initial state partons we have to perform mass factorization to absorb the initial state collinear divergences in the structure functions.

This forces one to first square both the virtual and the bremstralung diagrams in their regularized form and extract the divergent pieces by hand. After that one can again use the Monte Carlo approach. This method is cumbersome and involves manipulation of large number of terms making extension of the calculation to more complicated jet cross sections very difficult; furthermore the procedure has to be repeated for each new calculation.

Using these techniques in QCD calculations one can calculate the order α_s^2 jet cross sections in e^+e^- annihilation [4]. A special case occurs when a process is considered involving only one gluon. Here we get still an QED-like soft gluon factorization due to the absence of the three gluon vertex. Using this fact Owens and collaborators [5] were able to write next to leading order Monte Carlo's for several processes.

To proceed to more final state jets the soft gluon behavior has to be understood better and a similar kind of factorization of the soft gluon singularities has to be accomplished. This factorization was found by several authors [6] through the introduction of color ordered subamplitudes [2]. These gauge invariant subamplitudes have similar properties to QED amplitudes and exhibit the desired soft gluon behavior.

This was used in ref. [7] to study the behavior of final state partons and isolate the singular parts of phase space using the lorentz invariant minimal invariant mass s_{min} as the theoretical resolution parameter. If the invariant mass of a parton pair is smaller than the minimal invariant mass, its singular contribution is integrated out within the s_{min} cone. This is then added to the virtual corrections, thus canceling the final state soft and collinear divergences. The soft and collinear factors are universal, easy to evaluate in their regularized form and independent of the hard process. This makes the approach very general and applicable to any process involving final state partons.

To include initial state radiation we can cross the unphysical but finite all-outgoing parton cross section and perform an analytic continuation of the appropriate transcendental functions. However, in doing this we make an error since with this naive crossing, the initial state radiation is integrated over a final state phase space. This can easily be corrected by constructing a crossing function which is the appropriate initial state

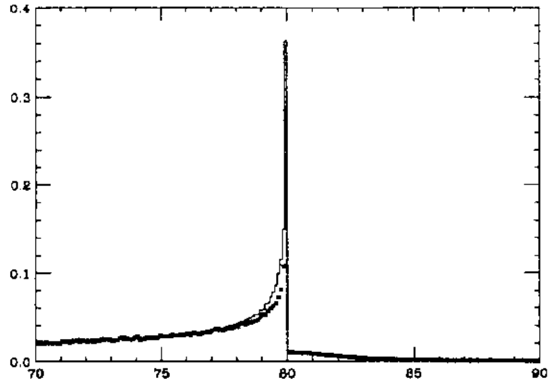

Figure 3: Transverse mass distribution in nb/GeV using the narrow width approximation for $p\bar{p} \rightarrow W^{\pm} + 0$ jets with the loose CDF cuts as described in the text. The lowest order result is shown as a histogram, while the next to leading order correction is shown as data points.

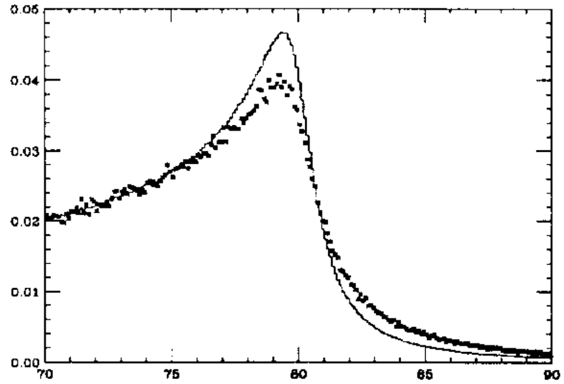

Figure 4: Transverse mass distribution in nb/GeV using the Breit Wigner resonance for $p\bar{p} \rightarrow W^{\pm} + 0$ jets with the loose CDF cuts as described in the text. The lowest order result is shown as a histogram, while the next to leading order correction is shown as data points.

collinear factor integrated over the correct phase space within the s_{min} cone minus the improperly added final state collinear factor. Only the crossing function is affected by mass factorization which then renders it finite. The crossing function is unique for each type of incoming parton, being a simple convolution of splitting functions with structure functions and is independent of the hard process. Once calculated, it can be used for all processes involving incoming partons regardless of the final state and without worrying about mass factorization. The crossing functions are analogous to structure functions in both appearance and use [8].

3 Applications

The use of these building blocks makes next to leading order QCD calculations very simple. First, we determine the all-outgoing parton cross section using the appropriate universal collinear and soft building blocks, giving a finite result [7]. Next we cross the partons we want in the initial state and add the finite crossing functions for these partons multiplied by the tree level cross section. Now we have the finite cross section within our resolution cut s_{min}. The necessary theoretical procedures, such as mass factorization, have been automatically included by using these building blocks [8].

All that remains to do is to add the hard radiation diagrams using a Monte Carlo. This will cancel the s_{min} dependence as is shown in figures 1 and 2 for $p\bar{p} \rightarrow \{W^{\pm} \rightarrow l^{\pm}\nu\} + 0, 1$ jets at Fermilab energies using the loose CDF jet cuts ($E_T(j) > 15$ GeV, $\Delta R(j,j) > 0.7$ and $|\eta(j)| < 2$) and lepton cuts ($E_T(l^{\pm}) > 20$ GeV, $E_T(\text{miss}) > 20$ GeV and $|\eta(l^{\pm})| < 1$), including all lepton correlations.

In figure 3 the transverse mass distribution is shown for the same cuts in the narrow width approximation, the W mass is chosen to be 80 GeV. The missing transverse momentum is calculated using the parton and charged lepton radiation in the appropriate rapidity ranges. These cuts illustrate the kind of results one can obtain with these type of calculations.

As can be seen from figure 3 the radiative effects reduce the W mass peak and a radiative tail above the W mass is generated through final state radiation giving the W boson momentum a transverse component.

In figure 4 the narrow width approximation is replaced by the Breit Wigner resonance with a width of 2 GeV. This leads to a smearing of the narrow width distribution, though the basic changes generated by the next to leading order corrections remain the same for both distributions. Note that including the Breit Wigner shifts the peak location somewhat below the W mass due to the convolution with the parton densities, both at leading order and next to leading order.

This distribution is important, because it is used in the W mass determination. As can be seen the location of the peak is not affected very much by the inclusion of the next to leading order corrections, while the downward slope above the W mass is changed considerably [9].

A more detailed phenomenology of the process $p\bar{p} \to W^{\pm}/Z + 0, 1$ jet at next to leading order, together with the crossing function techniques, can be found in ref. [8].

4 Conclusions

With the use of the soft/collinear factors and crossing functions the construction of the unresolved phase space is easy [7,8]. Only the finite virtual corrections remain to be calculated.

Note that by using the factorization properties we can cancel the divergences explicitly before specifying the hard processes. This avoids having to square the real graphs in n dimensions which is a great simplification over the standard methods used in QCD calculations. Also the finite virtual graphs can, with the necessary care, be evaluated in four dimensions. The method reveals the simple underlying structure of radiative corrections in QCD and stresses the analogy with QED through the use of the subamplitudes.

Once the virtual and soft/collinear contributions are calculated, one can add the hard phase space numerically allowing the numerical implementation of jet clustering, jet defining cuts and other cuts. This enables us to write flexible Monte Carlo's which can be used to compare with experimental data.

This method simplifies the calculation of the next to leading order corrections considerably and makes the calculation of processes such as $e^+e^- \to 4$ jets, its crossing $p\bar{p} \to W^{\pm}/Z + 2$ jets and $p\bar{p} \to 3$ jets at next to leading order possible. These multijet cross sections are important for the forthcoming experiment at LEP and Fermilab since the event rates are expected to be high and will be studied in great detail.

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